

Cavity-output-field control via interference effects

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We show how interference effects are responsible for manipulating the output electromagnetic field of an optical micro-resonator in the good-cavity limit. The system of interest consists in a moderately strongly pumped two-level emitter embedded in the optical cavity. When an additional weaker laser of the same frequency is pumping the combined system through one of the resonator's mirror then the output cavity electromagnetic field can be almost completely suppressed or enhanced. This is due to the interference among the scattered light by the strongly pumped atom into the cavity mode and the incident weaker laser field. The result applies to photonic crystal environments as well.

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I. INTRODUCTION

Present and future quantum technologies require various tools allowing for complete or partial control of the quantum mechanical interaction between light and matter. Therefore, quite significant amount of works are dedicated to this issue. Particularly, light interference is an widely investigated topic and, no doubts, its importance for various applications is enormous [1–5]. Due to quantum interference effects for instance elimination of spectral lines or complete cancellation of the spontaneous decay can occur. Spatial interference shows interesting features as well [2, 4, 5]. Furthermore, suppression of the resonance fluorescence in a lossless cavity was demonstrated in [6] whereas cavity-field-assisted atomic relaxation and suppression of resonance fluorescence at high intensities was shown in [7]. Inhibition of fluorescence in a squeezed vacuum was demonstrated in [8] while suppression of Bragg scattering by collective interference of spatially ordered atoms within a high-Q cavity mode was demonstrated, respectively, in [9]. On the other side, cavity-enhanced single-atom spontaneous emission was observed in Ref. [10] while suppression of spontaneous decay at optical frequencies was shown in [11]. The control of the spontaneous decay as well as of the resonance fluorescence is of particular interest for quantum computation processes [12] where, in addition, highly correlated photons are required [13]. Combining few coherent driving sources one can achieve a further degree of control of the atom's quantum dynamics. Actually, the bichromatic driving of single atoms was intensively investigated recently emphasizing interesting interference phenomena. In particular, the resonance fluorescence of a two-level atom in a strong bichromatic field was analyzed in [14] and the response of a two-level system to two strong fields was experimentally studied in [15], correspondingly. The decay of bichromatically driven atom in a cavity was in-

vestigated in [16]. Broadband high-resolution x-ray frequency combs were obtained via bichromatically pumping of three-level Λ -type atoms [17]. Moreover, bichromatic driving of a solid-state cavity quantum electrodynamics system was investigated in Ref. [18]. Finally, photonic crystal's influence on quantum dynamics of pumped few-level qubits was investigated in detail as well [19–21].

The above mentioned works may be of particular relevance in a quantum network [22, 23], for instance. Related systems have been already proven to act as an optical diode [24, 25] - an important ingredient in a quantum network. Since a precise control over system's properties is highly required in such a network, here, we investigate the feasibility of controlling the cavity output electromagnetic field in a system consisting of a moderately strongly pumped two-level emitter. If a second coherent driving is applied through one of the mirrors and perpendicular to the first laser-beam then the output cavity field can be almost completely inhibited in the good-cavity limit. Notice that the lasers are in resonance with the cavity mode frequency. We have found that the interference between the second weaker light beam and the light scattered by the two-level emitter into the cavity mode due to stronger pumping is responsible for the suppression effect. The destructive interference can be turned into constructive one (or viceversa) via varying the phase difference of the applied lasers. Furthermore, the inhibition requires the laser frequency to be out of atomic frequency resonance while for photonic crystals surroundings it can be even on resonance.

The article is organized as follows. In Sec. II we describe the analytical approach and the system of interest, while in Sec. III we analyze the obtained results. The summary is given in Sec. IV.

II. QUANTUM DYNAMICS OF A PUMPED TWO-LEVEL ATOM INSIDE A DRIVEN MICROCAVITY

The Hamiltonian describing a two-level atomic system having the transition frequency ω_0 and interacting with a

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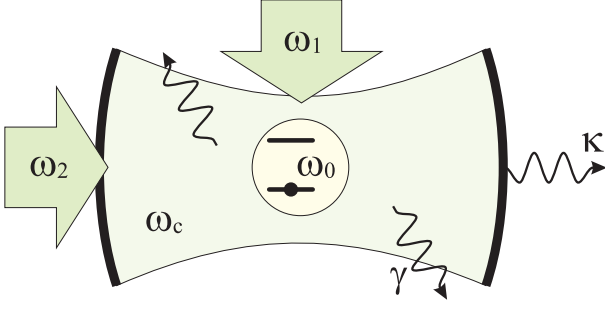


FIG. 1: (color online) The schematic of the model: A two-level emitter possessing the transition frequency ω_0 embedded in a single-mode (ω_c) micro-cavity is pumped with an intense laser field of frequency ω_1 . A second coherent source of frequency ω_2 is driving the entire system through one of the mirrors. γ is the single-atom spontaneous decay rate, while κ describes the cavity photon leaking rate, respectively.

strong coherent source of frequency ω_1 while embedded in a pumped micro-cavity of frequency ω_c , in a frame rotating at $\omega = \omega_1 = \omega_2$ (See Figure 1), is:

$$H = \hbar\Delta S_z + \hbar\delta a^\dagger a + \hbar g(a^\dagger S^- + a S^+) + \hbar\Omega(S^+ e^{i\phi_1} + S^- e^{-i\phi_1}) + \hbar\epsilon(a^\dagger e^{i\phi_2} + a e^{-i\phi_2}), \quad (1)$$

where $\Delta = \omega_0 - \omega$, and $\delta = \omega_c - \omega$. In the Hamiltonian (1) the components, in order of appearance, describe the atomic and the cavity free energies, the interaction of the two-level emitter with the micro-cavity mode and the atom's interaction with the first laser field with Ω being the corresponding Rabi frequency, and respectively, the interaction of the second driving field with the cavity mode with ϵ being proportional to the input laser field strength amplitude. The atomic bare-state operators $S^+ = |e\rangle\langle g|$ and $S^- = [S^+]^\dagger$ obey the commutation relations for su(2) algebra, i.e., $[S^+, S^-] = 2S_z$ and $[S_z, S^\pm] = \pm S^\pm$. Here, $S_z = (|e\rangle\langle e| - |g\rangle\langle g|)/2$ is the bare-state inversion operator. $|e\rangle$ and $|g\rangle$ are, respectively, the excited and ground state of the atom while a^\dagger and a are the creation and the annihilation operator of the electromagnetic field (EMF) in the resonator, and satisfy the standard bosonic commutation relations, namely, $[a, a^\dagger] = 1$, and $[a, a] = [a^\dagger, a^\dagger] = 0$ [26, 27]. $\{\phi_1, \phi_2\}$ are the corresponding phases of the coherent driving sources.

We shall describe our system using the laser-qubit semiclassical dressed-state formalism defined as [4]:

$$\begin{aligned} |+\rangle &= \sin\theta|g\rangle + \cos\theta|e\rangle, \\ |-\rangle &= \cos\theta|g\rangle - \sin\theta|e\rangle, \end{aligned} \quad (2)$$

with $\tan 2\theta = 2\Omega/\Delta$. Applying this transformation to (1) one arrives then at the following dressed-state Hamiltonian

$$H = H_0 + \hbar g(\cos^2\theta R^- - \sin^2\theta R^+)a^\dagger e^{-i\phi} + \hbar g(\cos^2\theta R^+ - \sin^2\theta R^-)a e^{i\phi}, \quad (3)$$

with

$$H_0 = \hbar\bar{\Omega}R_z + \hbar\delta a^\dagger a + \hbar\epsilon(a^\dagger + a) + \hbar R_z(g_0^* a^\dagger + g_0 a). \quad (4)$$

Here, $\bar{\Omega} = \sqrt{\Omega^2 + (\Delta/2)^2}$ while $g_0 = (g/2)\sin 2\theta e^{i\phi}$, $g_0^* = (g/2)\sin 2\theta e^{-i\phi}$ and $\phi = \phi_2 - \phi_1$. We employed also $S^\pm = \tilde{S}^\pm e^{\mp i\phi_1}$ and $a^\dagger = \tilde{a}^\dagger e^{-i\phi_2}$ with $a = [a^\dagger]^\dagger$ in the Hamiltonian (1) and dropped the tilde afterwards. The new quasi-spin operators, i.e., $R^+ = |+\rangle\langle -|$, $R^- = [R^+]^\dagger$ and $R_z = |+\rangle\langle +| - |-\rangle\langle -|$ are operating in the dressed-state picture. They obey the following commutation relations: $[R^+, R^-] = R_z$ and $[R_z, R^\pm] = \pm 2R^\pm$.

Considering that $\delta \ll \bar{\Omega}$ the last two terms in Eq. (3) can be ignored under the secular approximation. Therefore, the master equation describing the laser-dressed two-level atom inside a leaking pumped resonator and damped via the vacuum modes of the surrounding EMF reservoir is:

$$\begin{aligned} \frac{d}{dt}\rho(t) + \frac{i}{\hbar}[H_0, \rho] &= -\kappa[a^\dagger, a\rho] - \Gamma_0[R_z, R_z\rho] \\ &- \Gamma_+[R^+, R^-\rho] - \Gamma_-[R^-, R^+\rho] + H.c.. \end{aligned} \quad (5)$$

Here

$$\begin{aligned} \Gamma_0 &= (\gamma_0 \sin^2 2\theta + \gamma_d \cos^2 2\theta)/4, \\ \Gamma_+ &= \gamma_+ \cos^4 \theta + (\gamma_d/4) \sin^2 2\theta, \\ \Gamma_- &= \gamma_- \sin^4 \theta + (\gamma_d/4) \sin^2 2\theta. \end{aligned}$$

Respectively, $\gamma_0 = \pi \sum_k g_k^2 \delta(\omega_k - \omega)$ and $\gamma_\pm = \pi \sum_k g_k^2 \delta(\omega_k - \omega \mp 2\bar{\Omega})$ are the single-atom spontaneous decay rates being dependent on the density of modes, g_k , at the dressed-state frequencies $\{\omega, \omega \pm 2\bar{\Omega}\}$, while γ_d signifies the pure dephasing rate. In free space one has that $\gamma_0 = \gamma_\pm \equiv \gamma$. Note that the master equation (5) was obtained either under the intense-field condition or far off detuned field, i.e., it is valid when $\bar{\Omega} \equiv \sqrt{\Omega^2 + (\Delta/2)^2} \gg \{\delta, g, \epsilon, \Gamma_0, \Gamma_\pm\}$.

The equations of motion for the variables of interest can be easily obtained from the Master Equation (5). Therefore, the quantum dynamics is described by the following system of linear differential equations:

$$\begin{aligned} \frac{d}{dt}\langle a^\dagger a \rangle &= ig_0\langle R_z a \rangle + i\epsilon\langle a \rangle - ig_0^*\langle R_z a^\dagger \rangle - i\epsilon\langle a^\dagger \rangle \\ &- 2\kappa\langle a^\dagger a \rangle, \\ \frac{d}{dt}\langle R_z a \rangle &= -(\kappa + i\delta + 2\Gamma_+ + 2\Gamma_-)\langle R_z a \rangle \\ &- 2(\Gamma_+ - \Gamma_-)\langle a \rangle - i\epsilon\langle R_z \rangle - ig_0^*, \\ \frac{d}{dt}\langle R_z a^\dagger \rangle &= -(\kappa - i\delta + 2\Gamma_+ + 2\Gamma_-)\langle R_z a^\dagger \rangle \\ &- 2(\Gamma_+ - \Gamma_-)\langle a^\dagger \rangle + i\epsilon\langle R_z \rangle + ig_0, \\ \frac{d}{dt}\langle a \rangle &= -(\kappa + i\delta)\langle a \rangle - ig_0^*\langle R_z \rangle - i\epsilon, \\ \frac{d}{dt}\langle a^\dagger \rangle &= -(\kappa - i\delta)\langle a^\dagger \rangle + ig_0\langle R_z \rangle + i\epsilon, \\ \frac{d}{dt}\langle R_z \rangle &= -2(\Gamma_- + \Gamma_+)\langle R_z \rangle + 2(\Gamma_- - \Gamma_+). \end{aligned} \quad (6)$$

In the system of equations (6), we have used the trivial condition $R_z^2 = 1$ which is the case for a single-qubit system.

In the following Section, we shall discuss our results, i.e., the possibility of inhibiting the cavity output field in the steady-state via interference effects.

III. OUTPUT CAVITY FIELD CONTROL

One of the solutions of system (6) in the steady-state represents the mean-photon number in the micro-cavity mode, namely:

$$\langle a^\dagger a \rangle_s = A\epsilon^2 + B\epsilon + C. \quad (7)$$

For $\delta = 0$ and $\gamma_0 = \gamma_\pm \equiv \gamma$, the coefficients A , B and C are given by the following expressions:

$$\begin{aligned} A &= \frac{1}{\kappa^2}, \\ B &= -\frac{2g\gamma\Delta\Omega \cos \phi}{\kappa^2(\gamma\Delta^2 + 2(\gamma + \gamma_d)\Omega^2)}, \\ C &= \frac{g^2\Omega^2}{\kappa^2(\gamma\Delta^2 + 2(\gamma + \gamma_d)\Omega^2)} \\ &\times \frac{\gamma(\kappa + 2\gamma)\Delta^2 + 2\kappa(\gamma + \gamma_d)\Omega^2}{(\kappa + 2\gamma)\Delta^2 + 4(\kappa + \gamma + \gamma_d)\Omega^2}. \end{aligned} \quad (8)$$

Because of the quadratic dependence on ϵ , the minimum value of the mean-photon number is:

$$\langle a^\dagger a \rangle_s^{\min} = C - \frac{B^2}{4A}. \quad (9)$$

The above value is achieved at:

$$\epsilon^{\min} = -B/(2A).$$

Based on Eqs. (8) and (9) it follows that ϵ^{\min} is independent on $\{\kappa, \delta\}$ and its value does not exceed $\frac{g\sqrt{2}}{4}(1 + \gamma_d/\gamma)^{-1/2}$. The cavity output field, i.e., the number of photons escaping the cavity per second, can be evaluated via $\kappa\langle a^\dagger a \rangle_s$. Particularly, in Fig. (2) the minimum value of the steady-state mean-photon number is $\langle a^\dagger a \rangle_s^{\min} \approx 0.06$ and is achieved when $(\epsilon/\gamma)^{\min} \approx 0.54$. An explanation of the steady-state behaviors shown in Fig. (2) can be found if one represents the mean-photon number given by (7) as follows:

$$\begin{aligned} \langle a^\dagger a \rangle_s &= \frac{\epsilon}{\kappa^2} \{ \epsilon + |g_0| \langle R_z \rangle_s \cos \phi \} + \frac{|g_0|}{\kappa(\kappa + 2\Gamma_+ + 2\Gamma_-)} \\ &\times \{ |g_0| + \epsilon \langle R_z \rangle_s \cos \phi - 2(\Gamma_+ - \Gamma_-)(|g_0| \langle R_z \rangle_s \\ &+ \epsilon \cos \phi)/\kappa \}, \end{aligned} \quad (10)$$

where

$$\langle R_z \rangle_s = -(\Gamma_+ - \Gamma_-)/(\Gamma_+ + \Gamma_-).$$

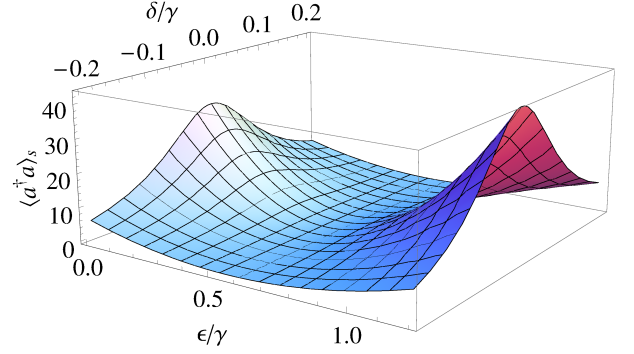


FIG. 2: (color online) The steady-state dependence of the micro-cavity mean photon number $\langle a^\dagger a \rangle_s$ versus the variables ϵ/γ and δ/γ . Other parameters are: $\gamma_d/\gamma = 0.01$, $\kappa/\gamma = 0.1$, $g/\gamma = 2$, $\Delta/\Omega = 3$ and $\phi = 0$.

From the above expression (10), one can see that for $\delta = 0$ the mean-photon number due to weaker external pumping of the cavity mode is proportional to ϵ^2 while that due to stronger driving of the two-level qubit to $|g_0|^2$, respectively. There is also a cross-contribution proportional to $\epsilon|g_0|\cos \phi$. All these terms demonstrate interference effects among the contributions due to two pumping lasers and, hence, the minima's nature in Fig. (2). In particular, for $\Gamma_+ \gg \Gamma_-$ one has from Eq. (10) that

$$\langle a^\dagger a \rangle_s \approx \frac{\epsilon^2}{\kappa^2} + \frac{|g_0|^2}{\kappa^2} - \frac{2\epsilon|g_0|}{\kappa^2} \cos \phi, \quad (11)$$

while for $\Gamma_- \gg \Gamma_+$ we have, respectively,

$$\langle a^\dagger a \rangle_s \approx \frac{\epsilon^2}{\kappa^2} + \frac{|g_0|^2}{\kappa^2} + \frac{2\epsilon|g_0|}{\kappa^2} \cos \phi. \quad (12)$$

Thus, indeed, the output field suppression (or enhancement) occurs because of the interference effect taking place among the fraction of light $|g_0|^2/\kappa^2$ scattered by the atom into the cavity mode due to stronger pumping by the first laser beam and, respectively, the photon field of the second weaker laser field characterized by ϵ^2/κ^2 (see, also, Figure 1). Furthermore, the nature of destructive or constructive interference can be understood as follows: In the dressed-state picture both lasers are simultaneously in resonance with the dressed-state transitions $|+\rangle \leftrightarrow |+\rangle$ and $|-\rangle \leftrightarrow |-\rangle$ and, hence, different signs in front of the last term in Eqs. (11,12). Actually, if $\Gamma_+ \gg \Gamma_-$ the atom is located on the lower dressed-state $|-\rangle$ while it resides on the higher dressed-state $|+\rangle$ when $\Gamma_- \gg \Gamma_+$. However, the destructive interference can be turned into constructive one (or viceversa) via varying the phase difference ϕ (see Eq. 11 and Eq. 12). This allows a better control of the output cavity field. Notice that on resonance, i.e., $\Delta = 0$, the inhibition effects are absent when $\gamma_+ = \gamma_-$ because $\Gamma_+ = \Gamma_-$ resulting in $\langle R_z \rangle_s = 0$ (see Eq. 10). Also, one can obtain small values for $\langle a^\dagger a \rangle_s$ if $\kappa > \gamma$. However, in this case we are in the

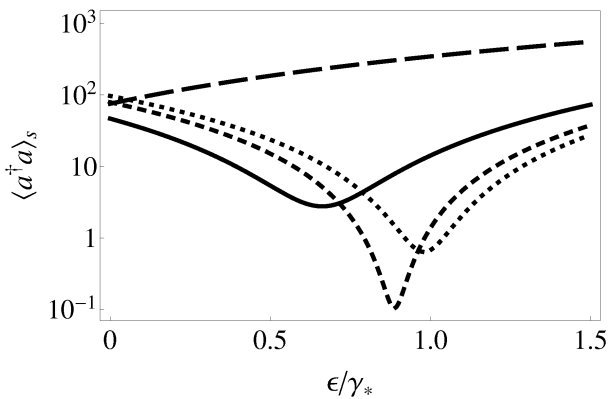


FIG. 3: The steady-state dependences of the mean-photon number $\langle a^\dagger a \rangle_s$ as a function of ϵ/γ_* . The solid-line is for $\gamma_* \equiv \gamma_+ = \gamma_-$, while the long-dashed curve stands for $\gamma_* = \gamma_-$ and $\gamma_+ \rightarrow 0$. Further, the short-dashed line is for $\gamma_* = \gamma_+$ and $\gamma_- \rightarrow 0$, whereas the dotted curve corresponds to $\Delta = \delta = 0$. Other parameters are the same as in Fig. (2) with $\Delta/\Omega = 1$ and $\delta/\gamma_* = 0$.

bad-cavity limit and, therefore, lower values for mean-photon number or even zero are expected [13, 26, 28]. Thus, in contrary, the cavity output field suppression reported here occurs in the good-cavity limit, i.e., when $\gamma > \kappa$ and $g > \{\kappa, \gamma\}$.

Apart from the dependence of the expression (10) on the parameters $\{\epsilon, \kappa, \phi, |g_0|\}$ it depends also on the generalized dressed-state decay rates Γ_\pm . These decay rates can be modified either by varying the detuning Δ in free-space with $\gamma_+ = \gamma_-$ or via modification of the density of modes at the dressed-state frequencies $\omega \pm 2\bar{\Omega}$ and, consequently, $\gamma_+ \neq \gamma_-$ which is a typical situation in photonic crystal environments [19–21], for instance. In particular, one can have also the situation when $\gamma_+ \gg \gamma_-$ or $\gamma_- \gg \gamma_+$. Figure (3) shows the mean-photon numbers obtained with the help of the expression (10) when the two-level emitter is located inside a microscopic cavity engineered in a photonic crystal material. In this case, the output cavity field can be suppressed even on atom-

laser frequency resonance, i.e. when $\Delta = 0$ (see the dotted curve), because $\gamma_+ \neq \gamma_-$ and the population will be distributed unequally among the dressed states while

$$\langle R_z \rangle_s = \frac{\gamma_- - \gamma_+}{\gamma_- + \gamma_+ + 2\gamma_d} \neq 0, \quad \text{if } \Delta = 0.$$

Negative values for the dressed-state inversion with $\phi = 0$ lead to cavity output field suppression (see Fig. 3 and Eq. 11). For the sake of comparison, the solid curve stands for ordinary vacuum-cavity environments. Thus, finalizing, we have shown here how the output cavity field can be minimized due to interference effects.

IV. SUMMARY

Summarizing, we have demonstrated the feasibility of cavity output field control via interference effects. The system of interest is formed from a strongly pumped two-level atom placed in an optical micro-resonator. A second weak laser being in resonance with the cavity mode frequency is probing the whole system through one of the cavity's mirror. Consequently, interference effects occur among the light scattered in the cavity mode by the strongly pumped atom and the incident weaker laser field leading to output cavity field inhibition or enhancement. Furthermore, the destructive interference can be turned into constructive one (or viceversa) via varying the phase difference of the applied lasers providing in this way a better control over output electromagnetic field. The idea works for photonic crystal environments as well.

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